Modeling Markets, Pandemics, and Peace: The Mathematics of Multi-Agent Systems

Lecture 6

Complex systems science

MIT HSSP
August 13th, 2022 – Starting 1:05

Recap: network analysis

Analyzing network structure can help us:

1) Identify the most influential agents

- Power of the Medici family

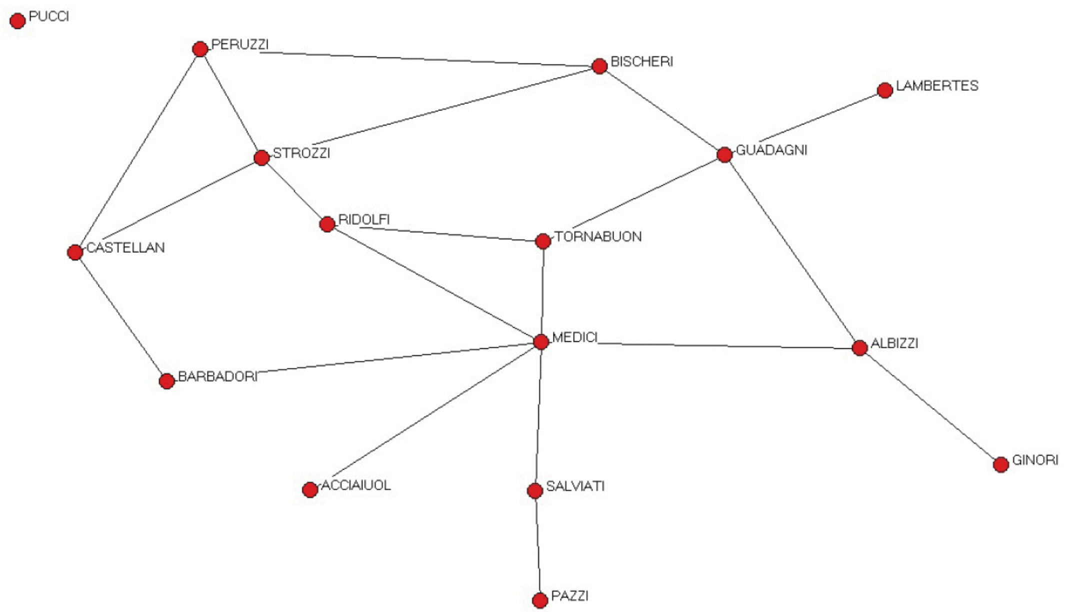
2) Study properties of network formation

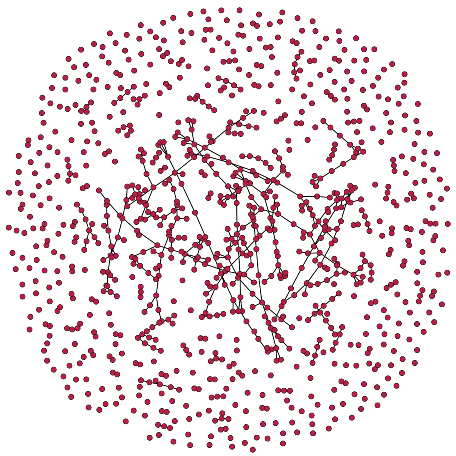
- Erdos-Reyni random graphs
- Preferential attachment
- Strategic network formation

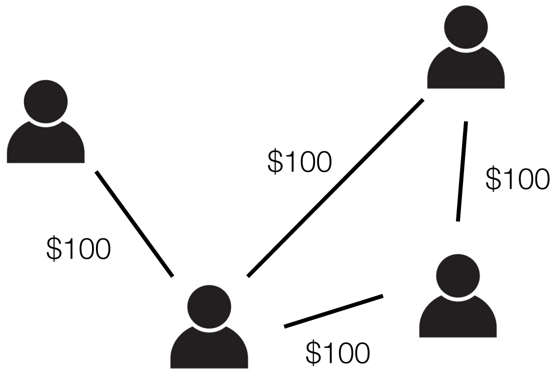
3) Predict outcomes of multi-agent games

- Routing games
- Bargaining games

… and much more!







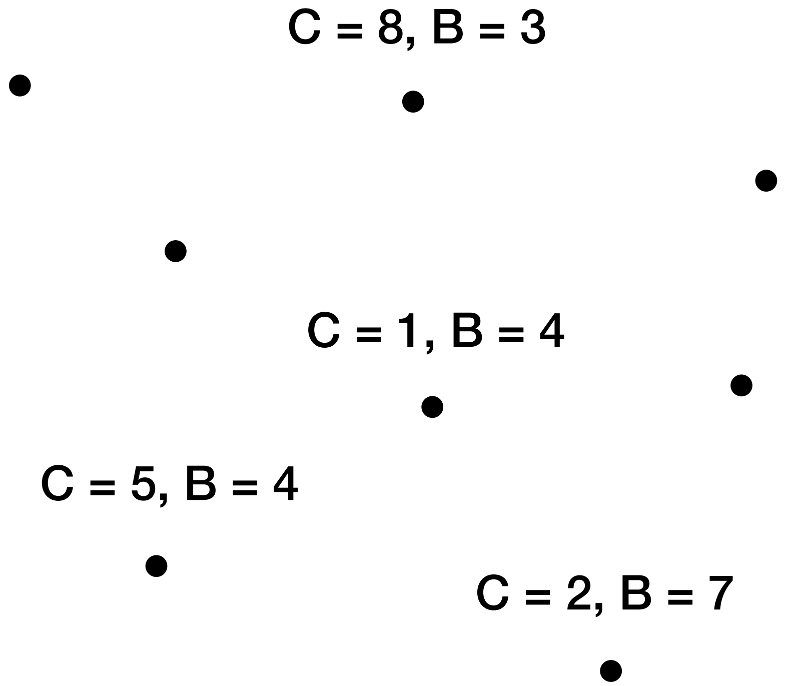
What makes a successful social movement?

*See Threshold Models of Collective Behavior by Mark Granovetter (1978)

Protest if benefits > costs such that it is worth it for a person to take to the streets

Costs = 5, Benefits = 4 → Stay at home

Costs = 2, Benefits = 3 → Protest



What makes a successful social movement?

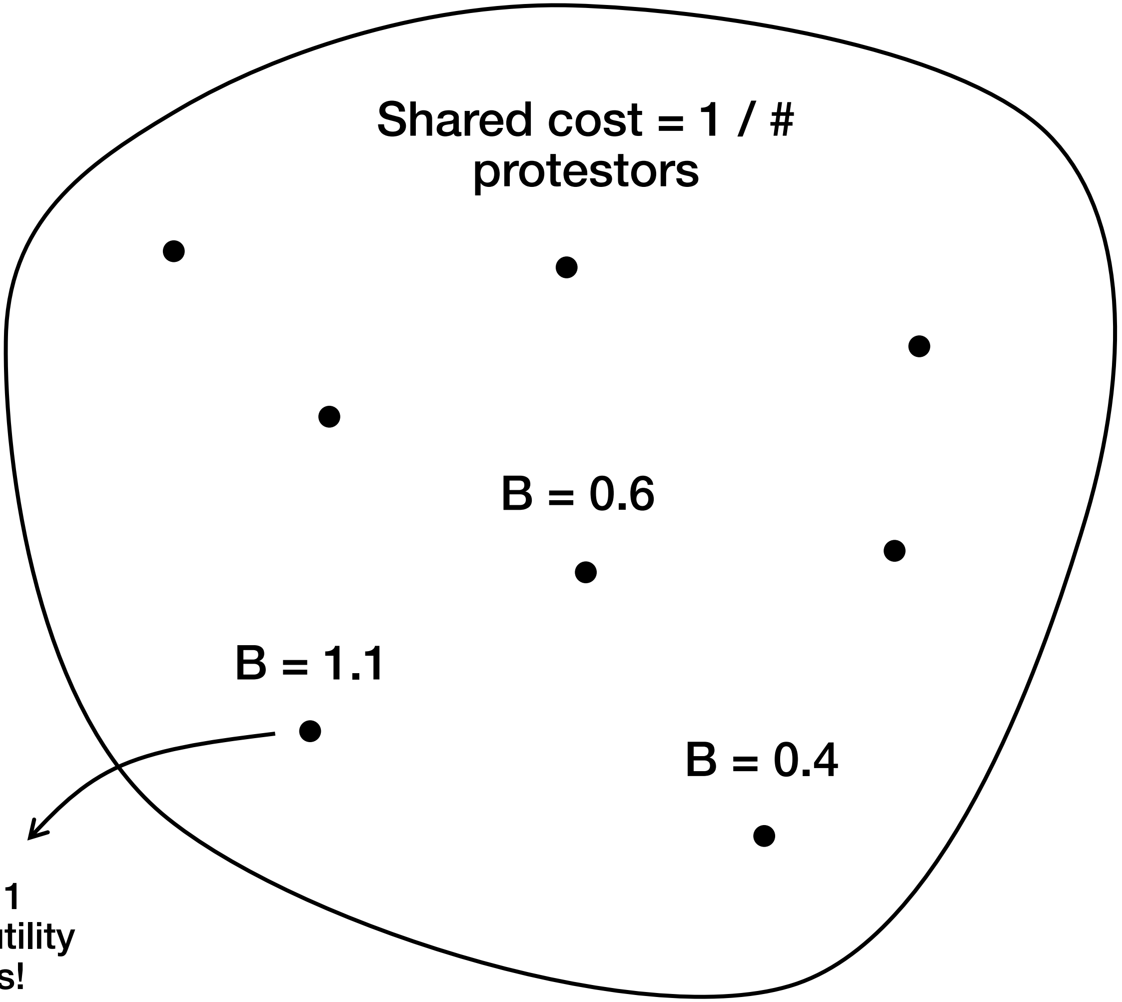
*See Threshold Models of Collective Behavior by Mark Granovetter (1978)

Protest if benefits > costs such that it is worth it for a person to take to the streets

Costs = 5, Benefits = 4 → Stay at home

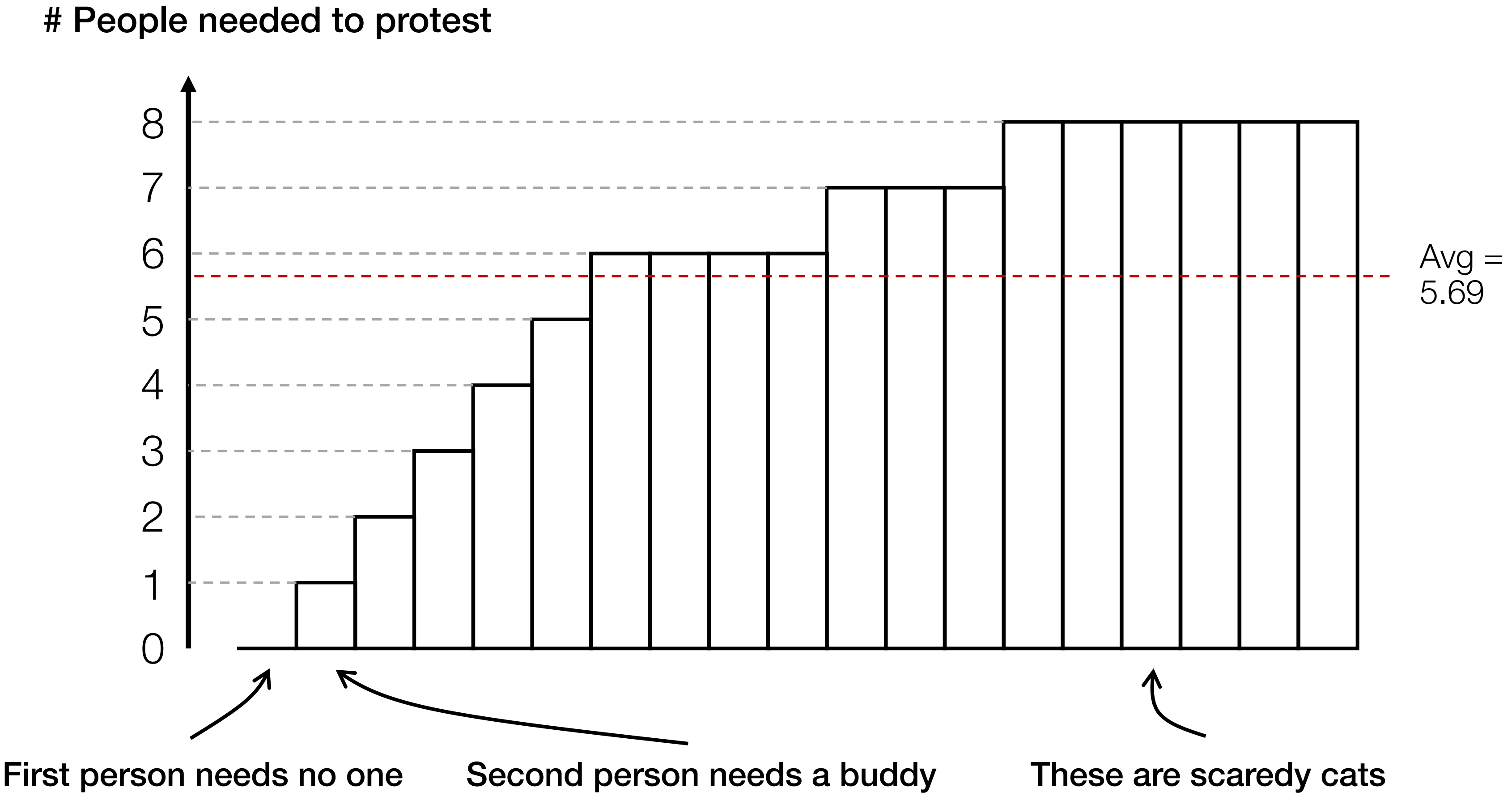
Costs = 2, Benefits = 3 → Protest

What if people's cost depended on one another, i.e., instead of looking at isolated agents, we make them **interact**?



Threshold models of protest

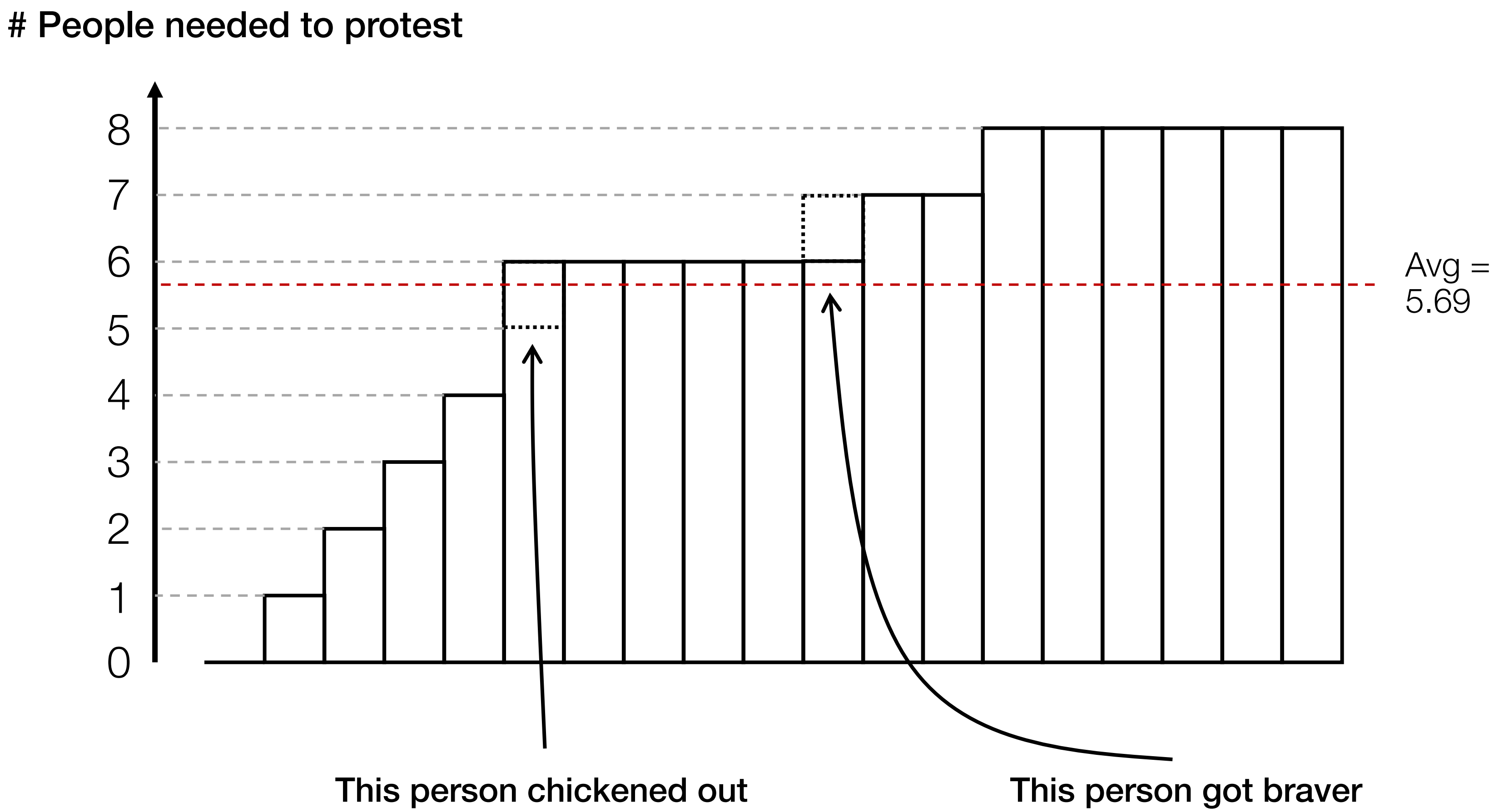
A threshold is the minimum # people someone needs to see on the streets before they decide to protest. Each person can have a different threshold.



Case 1: everyone ends up taking to the streets

Threshold models of protest

A threshold is the minimum # people someone needs to see on the streets before they decide to protest. Each person can have a different threshold.



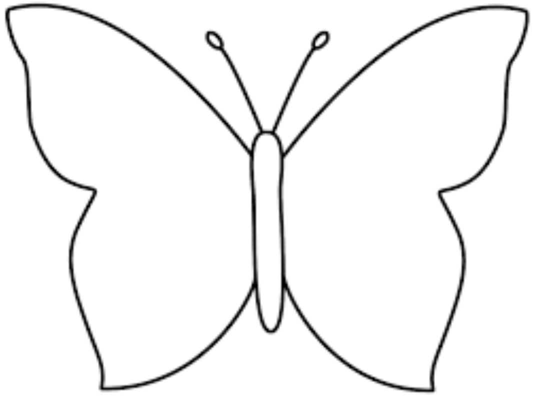
Case 1: everyone ends up taking to the streets

Case 2: the distribution is changed slightly, but the chain of dominos is broken!

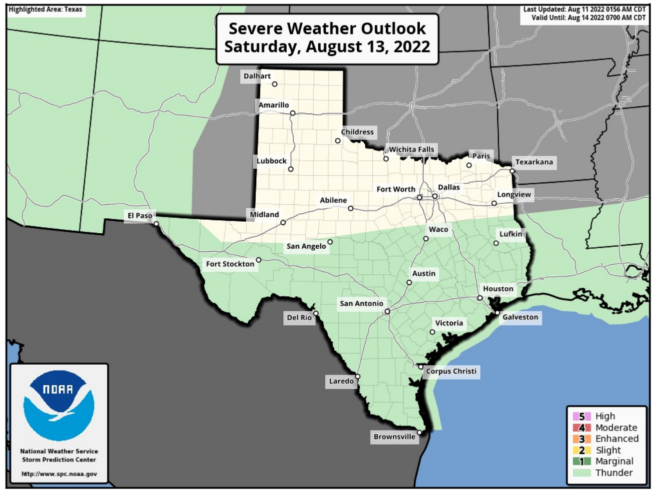
Note that in both cases, the average preference of the individuals are identical, but we have drastically different outcomes.

The butterfly effect

Systems whose components strongly interact nonlinearly exhibit
sensitive dependence on initial conditions



?



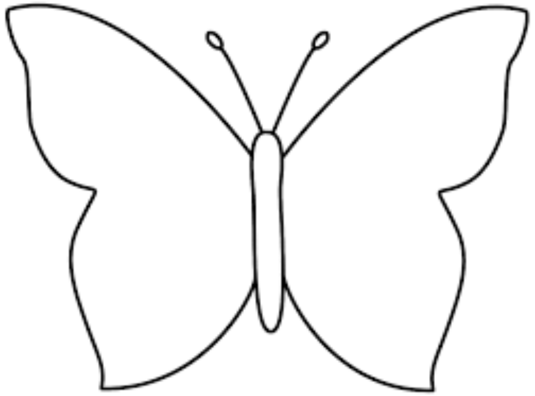
Small changes in the initial state

Large differences at a later state

Can a butterfly in Brazil cause a Tornado in Texas?

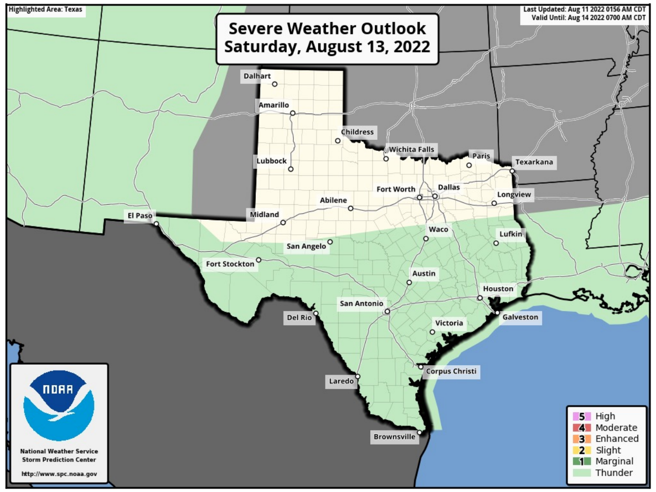
The butterfly effect

Systems whose components strongly interact nonlinearly exhibit
sensitive dependence on initial conditions



Small changes in the
initial state

Yes!



Large differences at a
later state

Can a butterfly in Brazil cause a Tornado in Texas?

A double pendulum

A double pendulum is an example of a strongly interacting nonlinear system

Two components coupled nonlinearly
to each other

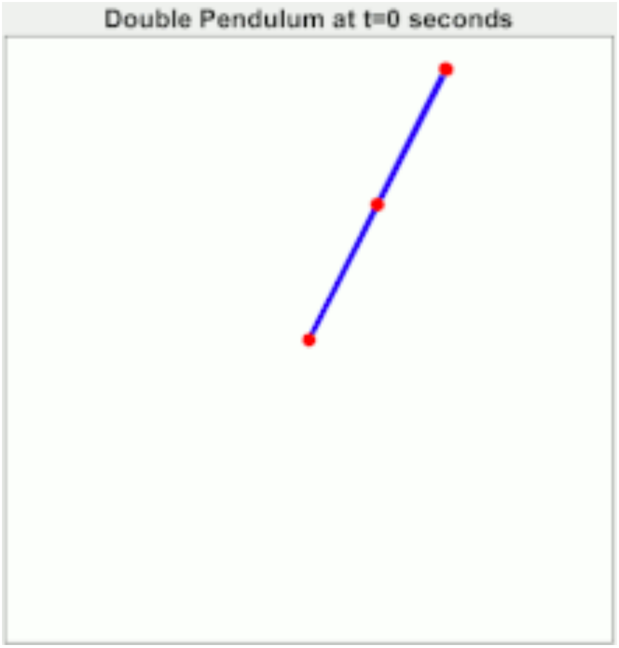
Chaotic motion
&
Sensitive dependence on initial conditions

$$x_1 = \frac{l}{2}\sin\theta_1 \qquad x_2 = l\left(\sin\theta_1 + \tfrac{1}{2}\sin\theta_2\right)$$

$$y_1 = -\frac{l}{2}\cos\theta_1 \qquad y_2 = -l\left(\cos\theta_1 + \tfrac{1}{2}\cos\theta_2\right)$$

Three double pendulums

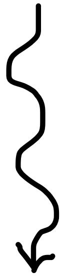
A double pendulum is an example of a strongly interacting nonlinear system



$$x_1 = \frac{l}{2} \sin\theta_1 \qquad x_2 = l\left(\sin\theta_1 + \tfrac{1}{2}\sin\theta_2\right)$$

$$y_1 = -\frac{l}{2} \cos\theta_1 \qquad y_2 = -l\left(\cos\theta_1 + \tfrac{1}{2}\cos\theta_2\right)$$

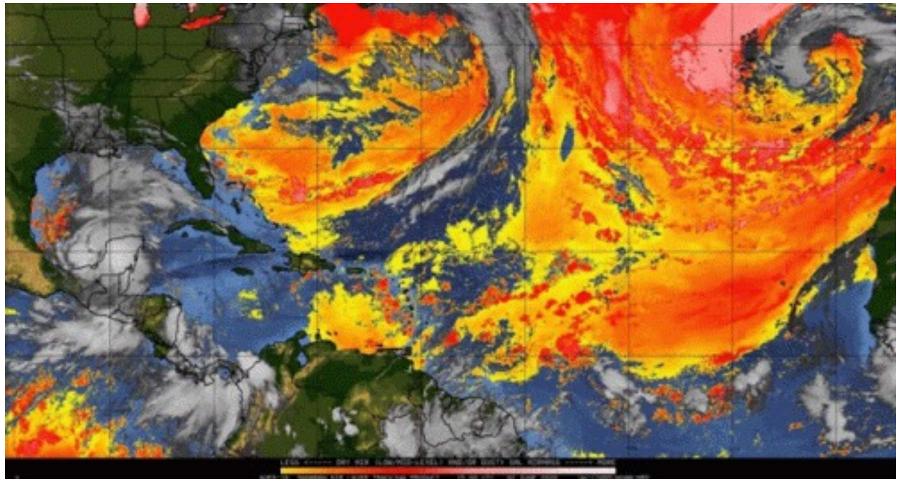
Two components coupled nonlinearly
to each other



Chaotic motion
&
Sensitive dependence on initial conditions

The weather

The weather is a more sophisticated example of a strongly interacting nonlinear system



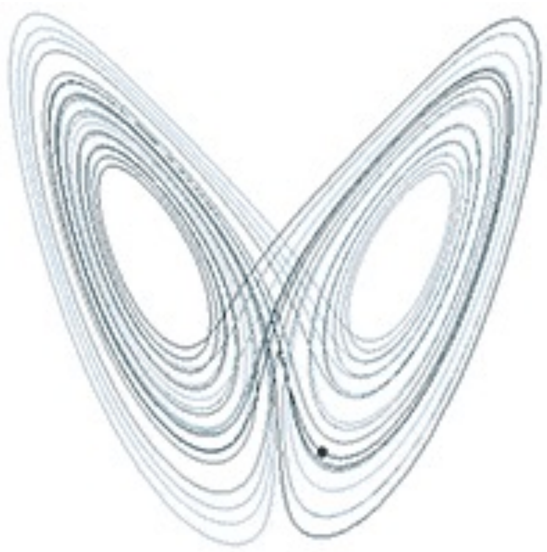
Lorenz equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

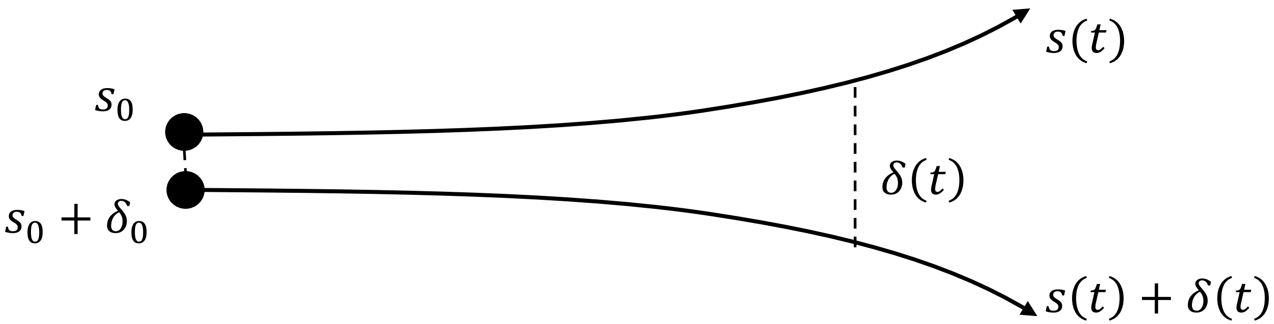
Sample solution



The Lorenz attractor

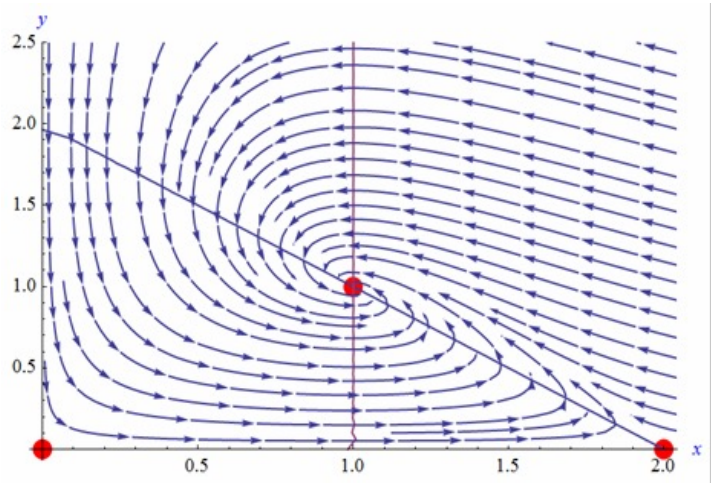
Measuring sensitivity

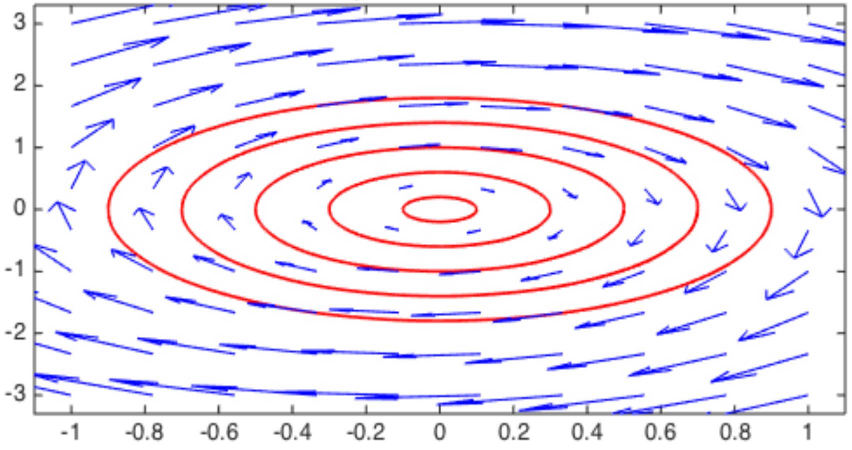
Lyapunov exponents characterize the rate of divergence of infinitesimally close trajectories

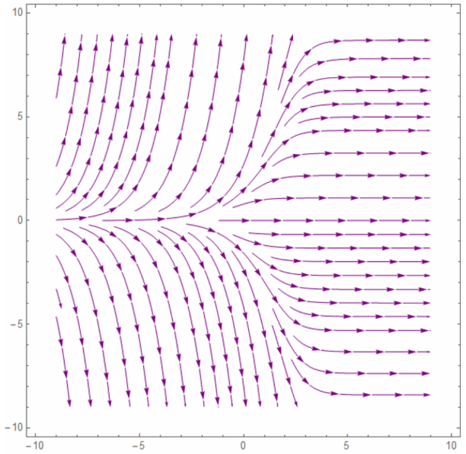


$$\delta(t) = \delta_0 e^{\lambda t}$$

λ = "Lyapunov exponent"

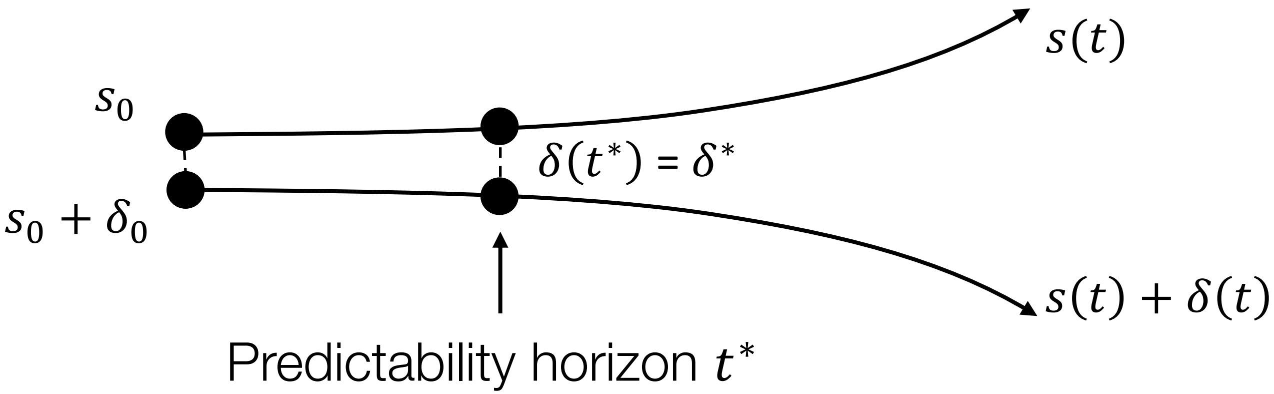






Predicting the weather

Can we predict the weather 100 days in advance?



$$\delta(t) = \delta_0 e^{\lambda t}$$

λ = "Lyapunov exponent"

Time beyond which you can't reliability predict:

$$t \approx \frac{1}{\lambda} \ln\left(\frac{\delta^*}{\delta_0}\right)$$

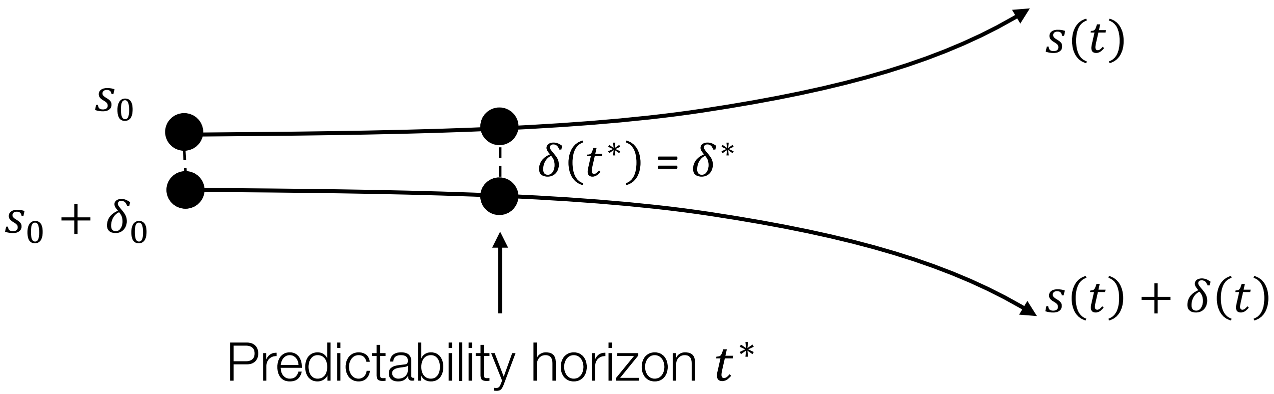
t = "Lyapunov time"

Let's say we can predict whether 10 days in advance (i.e. $\mathrm{t}^* = 10$). What if we wanted $\mathrm{t}^* = 100$?

For $\mathrm{t}^* \rightarrow 10\mathrm{t}^*$, we need $\delta_0 \rightarrow \frac{1}{e^{10}}\,\delta_0 \approx \frac{1}{22026}\,\delta_0$!

Predicting the weather

Can we predict the weather 100 days in advance?

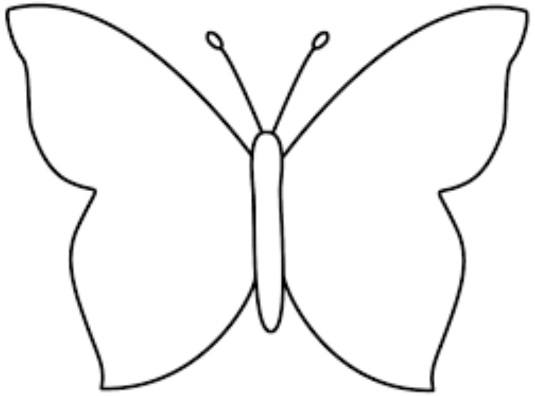


Let's say we can predict whether 10 days in advance (i.e. $\mathrm{t}^* = 10$). What if we wanted $\mathrm{t}^* = 100$?

For $\mathrm{t}^* \rightarrow 10\mathrm{t}^*$, we need $\delta_0 \rightarrow \frac{1}{e^{10}} \delta_0 \approx \frac{1}{22026} \delta_0$!

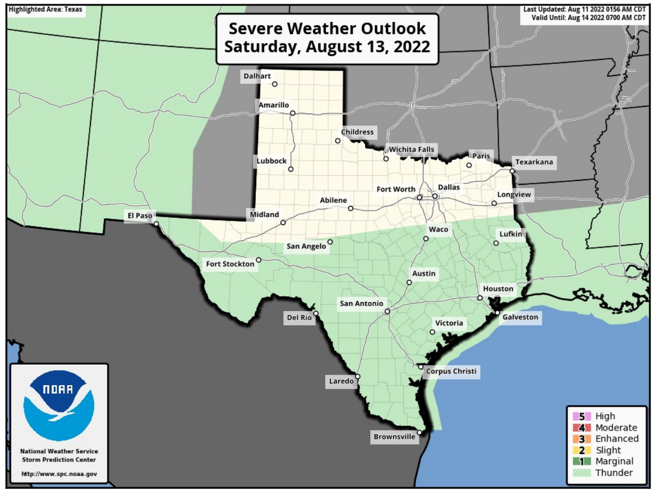
You can change it though! (if you look far enough ahead)

The butterfly effect



Small changes in the initial state

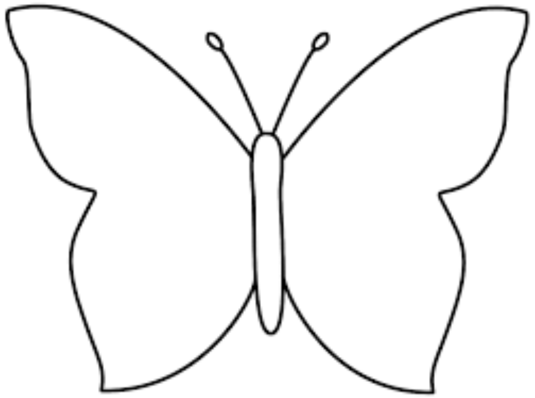
Yes!



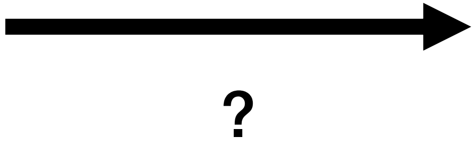
Large differences at a later state

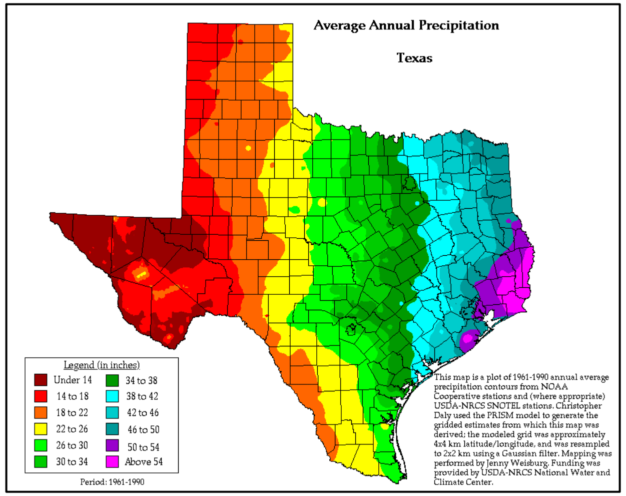
Can a butterfly in Brazil cause a Tornado in Texas?

The butterfly effect of climate?



Small changes in the initial state





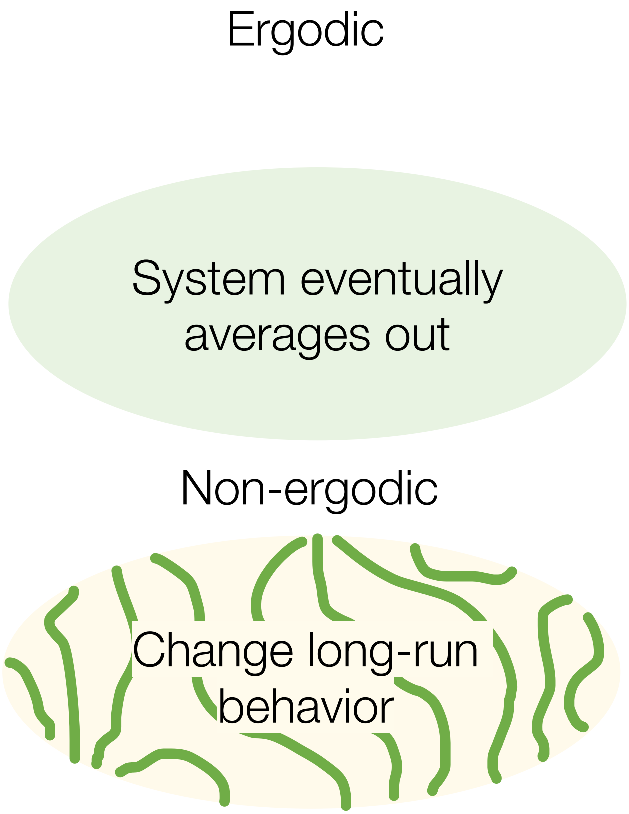
Long-run statistics

affect the climate

Can a butterfly in Brazil ~~cause a Tornado~~ in Texas?

Ergodicity

A system is ergodic if it eventually visits every possible state



Ergodicity

A system is ergodic if it eventually visits every possible state

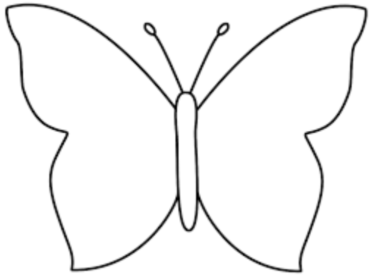
Ergodic

Climate eventually averages out

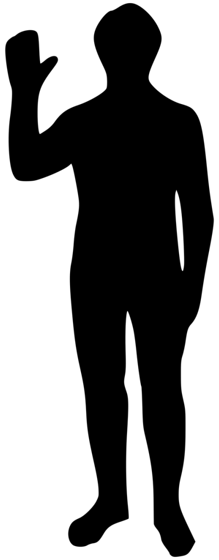
Non-ergodic

Solve climate change!

Can we change the world?



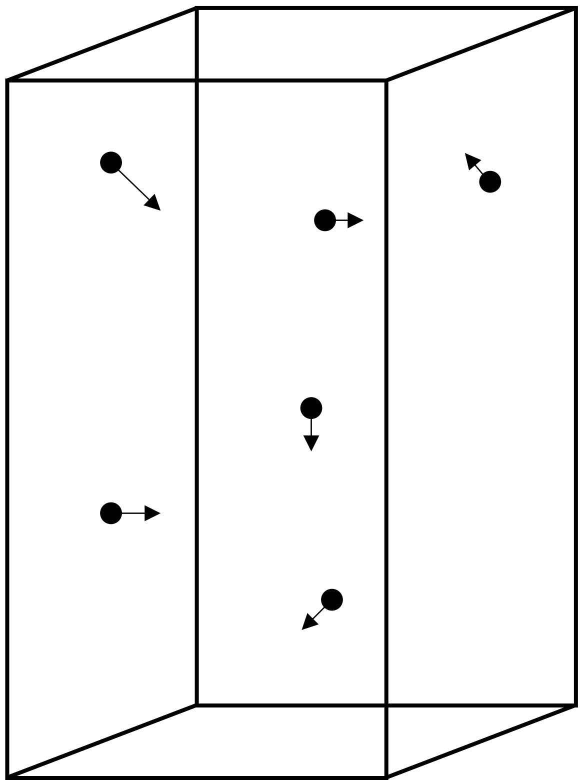
She’s a 10 but she can’t
solve climate change



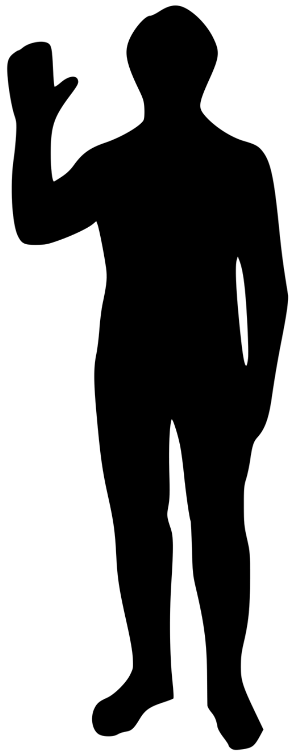
What about
you?

Which is more complex?

Complexity of a system = amount of information needed to describe its behavior.



12000 moles of ideal gas



A human person

How many bits of information is needed to completely describe the system?

$12000 \text{ moles} \sim 7\times10^{27} \text{atoms}$

Assuming the velocity and position of each gas molecule are described by a 256-bit floats:

$2.2\times10^{17} \text{ TB} \sim 2 \text{ trillion internets of data}$

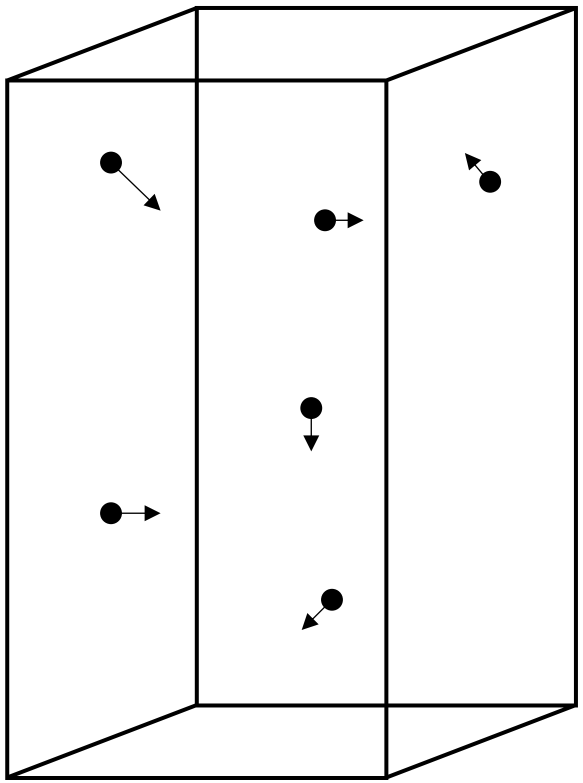
However, the human has a lot of order/repeated patterns, so we can compress that information to be a lot less!

Is the box of gas actually more complex?

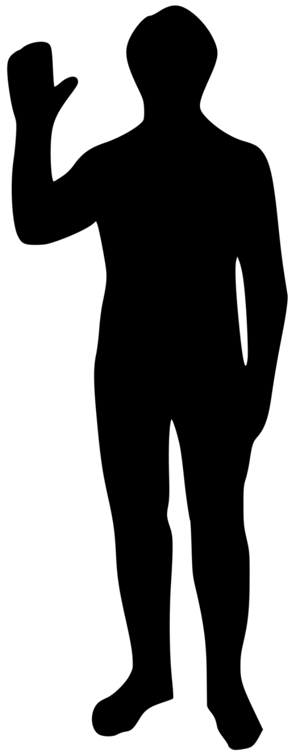
See An Introduction to Complex Systems Science and Its Applications by Alexander Siegenfeld and Yaneer Bar-Yam

Which is more complex?

Complexity of a system = amount of information needed to describe its behavior.



12000 moles of ideal gas



A human person

Not so fast!

We perceive the box of gas as being less complex because it doesn't have meaningful structure, while humans do.

In other words, due to **ergodicity**, the exact positions of the gas molecules actually doesn't matter at our level of aggregation, but the structure of the human certainly does.

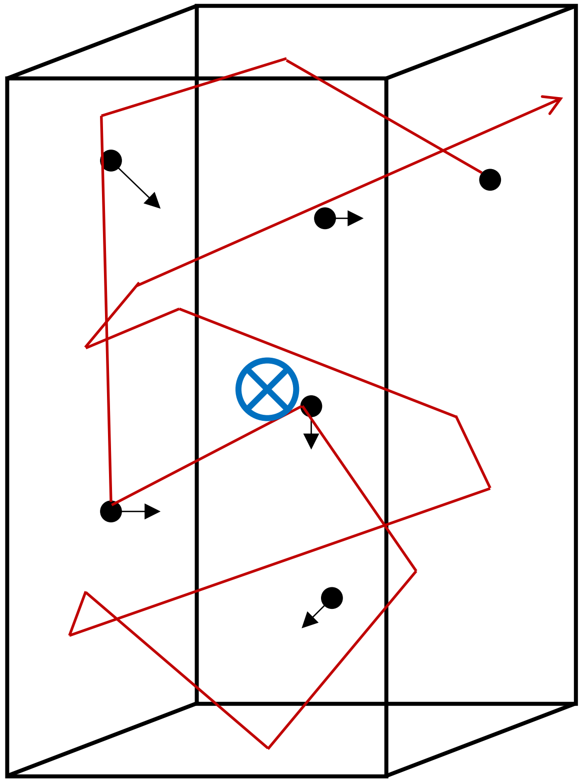
For us, the box of gas can be described by just three parameters!

Pressure (P)
Volume (V)
Temperature (T)

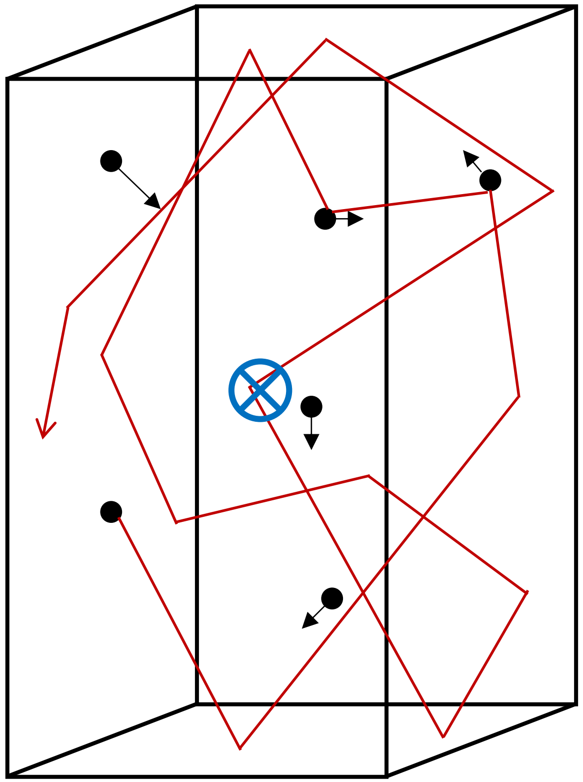
See An Introduction to Complex Systems Science and Its Applications by Alexander Siegenfeld and Yaneer Bar-Yam

Ergodicity saves the day

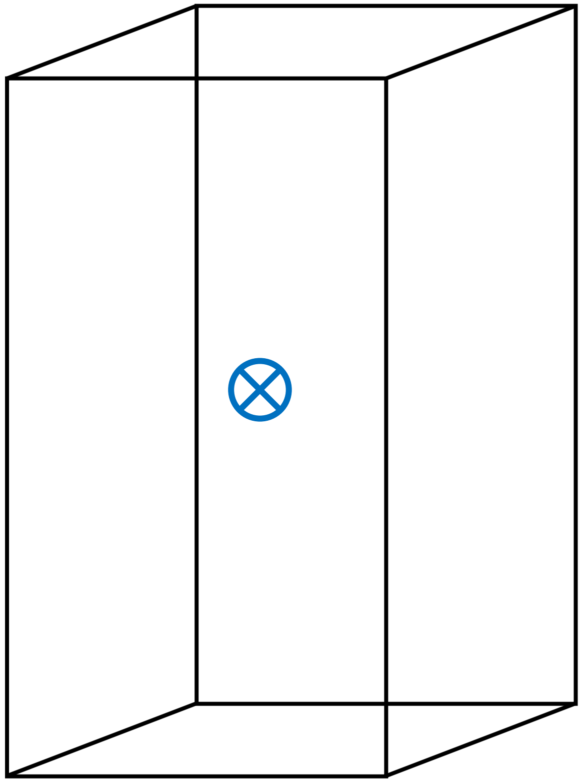
The large-scale behavior of the system can be adequately described by a few variables because statistical fluctuations average out over time.



Trajectory of particle A

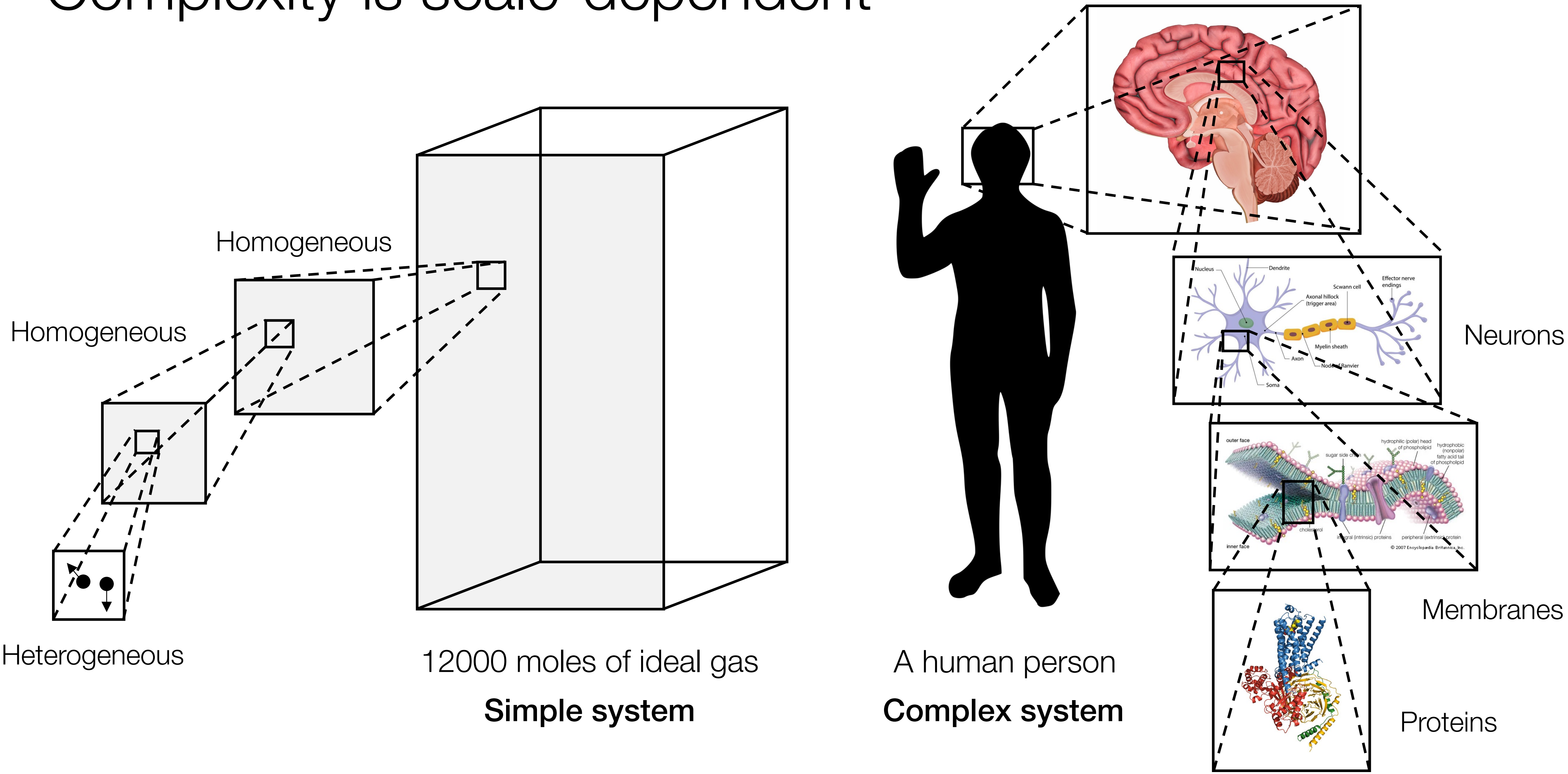


Trajectory of particle B



Trajectory of any (representative) particle

Complexity is scale-dependent

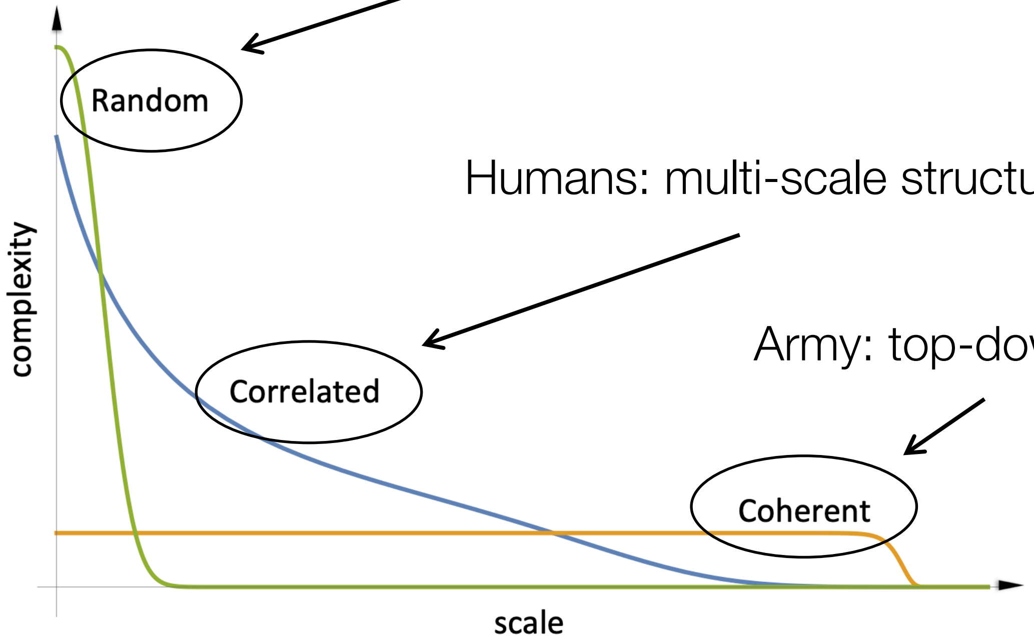


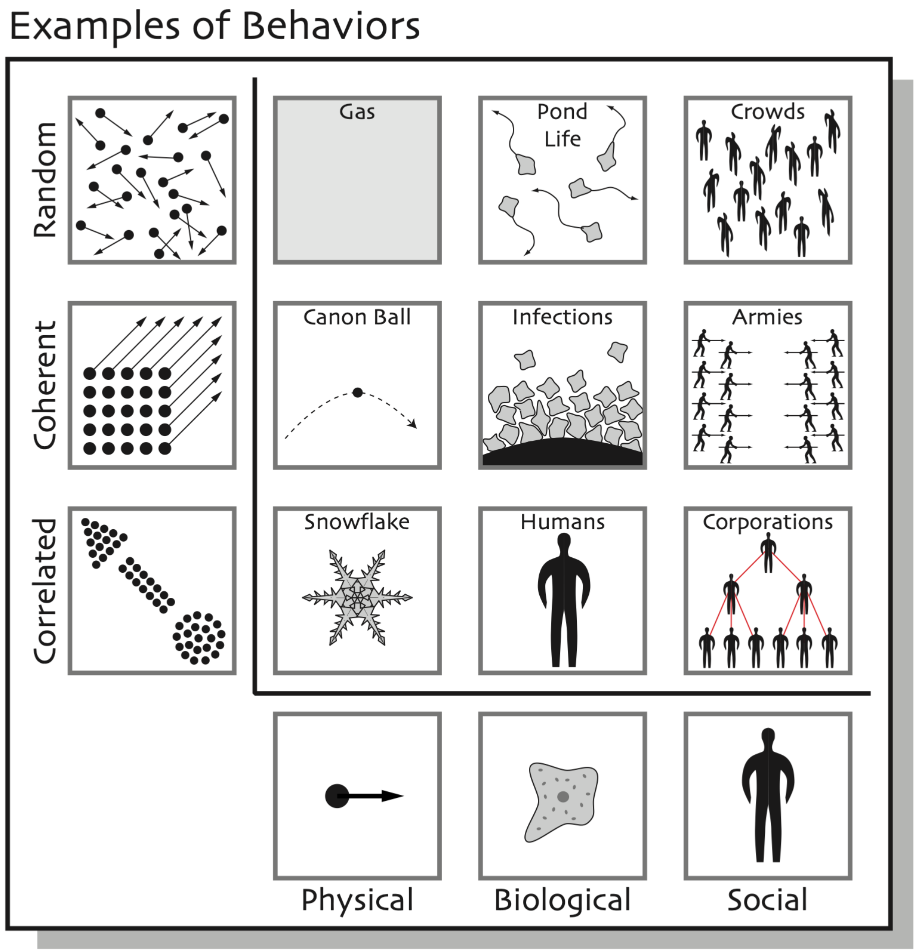
Complexity profiles

Gas: a lot of complexity at a tiny scale, but little at larger scales

Humans: multi-scale structures

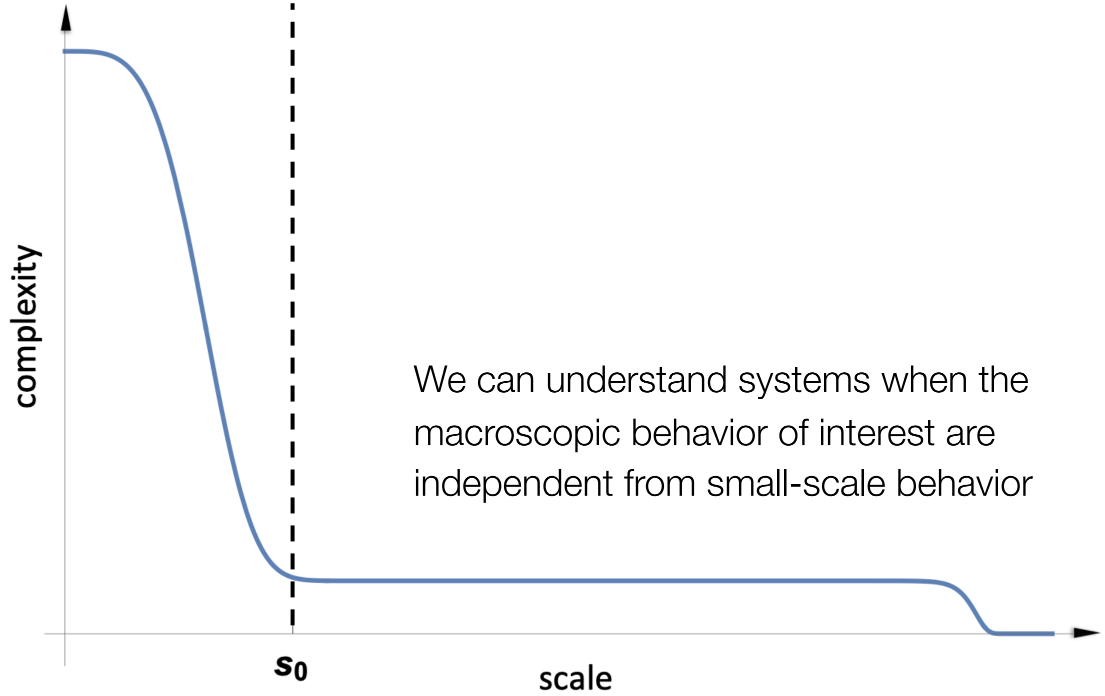
Army: top-down control



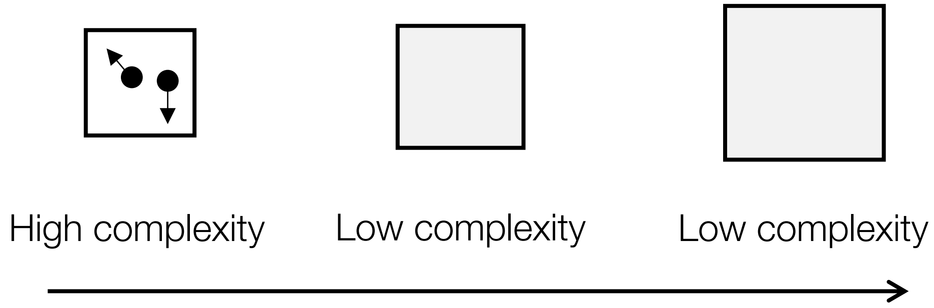


See An Introduction to Complex Systems Science and Its Applications by Alexander Siegenfeld and Yaneer Bar-Yam

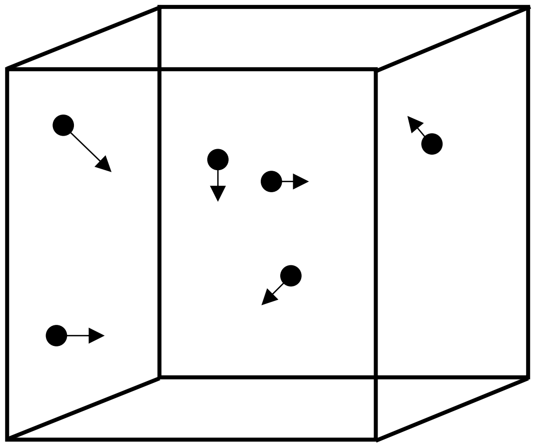
Separation of scale



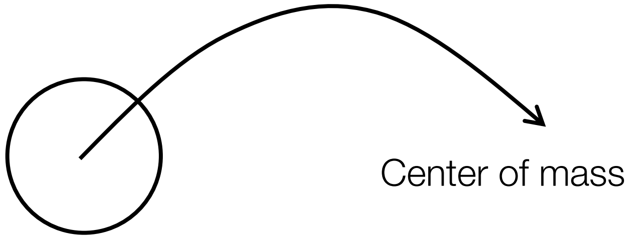
We can understand systems when the macroscopic behavior of interest are independent from small-scale behavior



[1] Siegenfeld and Bar-Yam (2020)



The gas can be described by a few parameters because trajectories are ergodic: they all average out to a simple behavior at a large scale!



Breakdown of mean-field descriptions

Because of interactions between components, simply describing systems by their average does not suffice. The microscopic details matter!

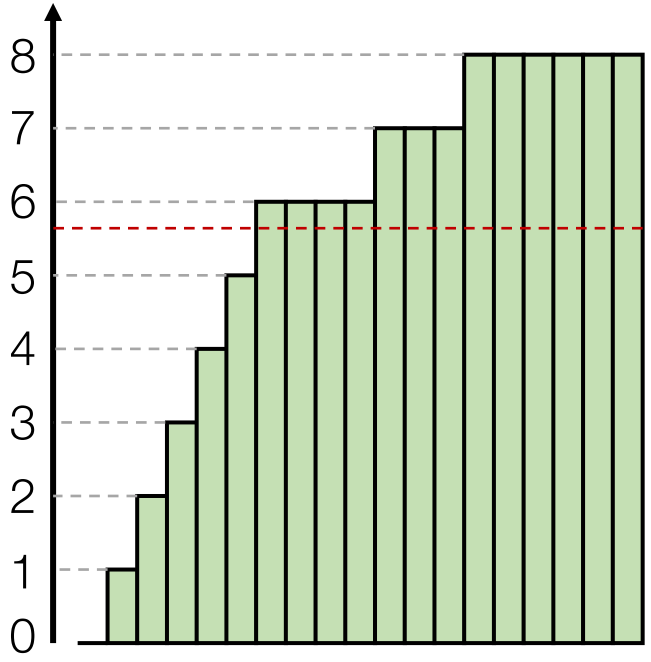
Mean-field/traditional cost/benefit analysis

Aggregate threshold of 5.69

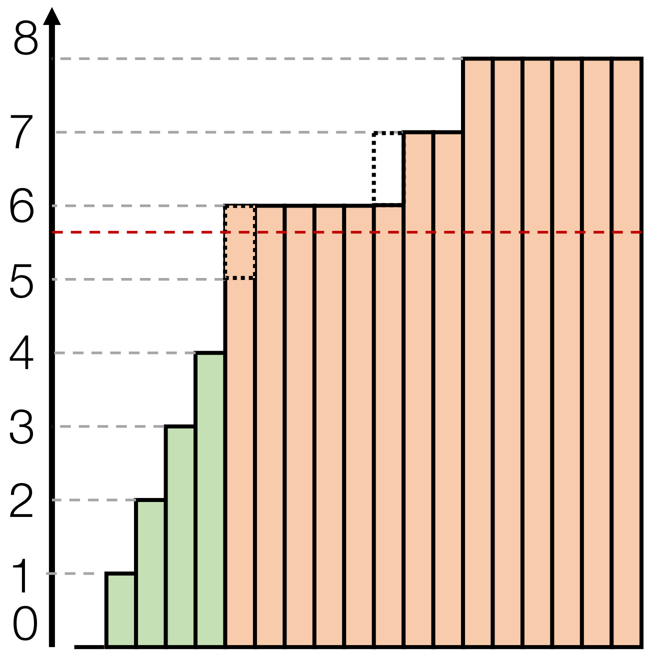
- Stay at home if benefits > threshold
- Protest if threshold > benefit

People either go protest or not at all, the details of the distribution does not matter.

Strongly-interacting systems: microscopic details matter!



Average threshold = 5.69

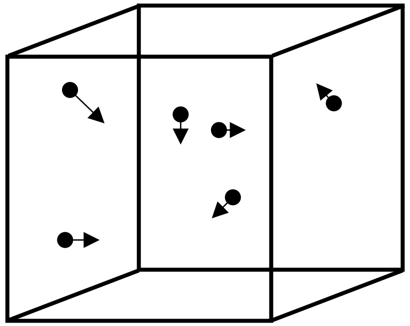


Average threshold = 5.69

We’ve seen mean-field theories before

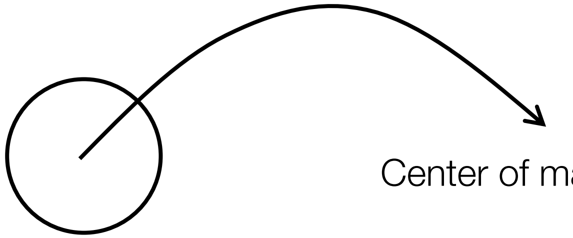
Mean-field descriptions underly many theories in natural and social science. They generally work quite well, but can break down in complex systems.

Statistical physics



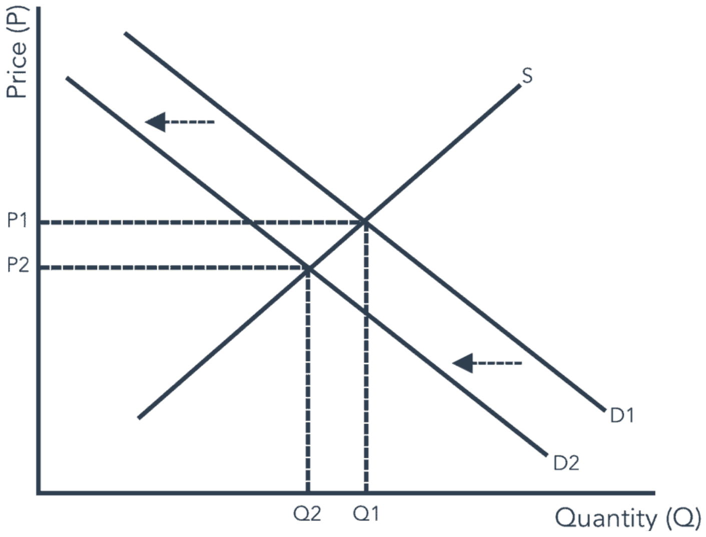
Pressure, Volume, Temperature

Newtonian physics



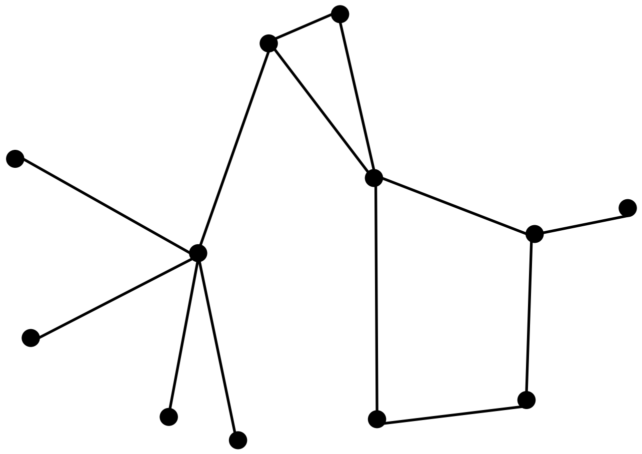
Center of mass

Mean-field games (markets)



People interact with the aggregate forces of supply and demand. We can define a “representative agent.”

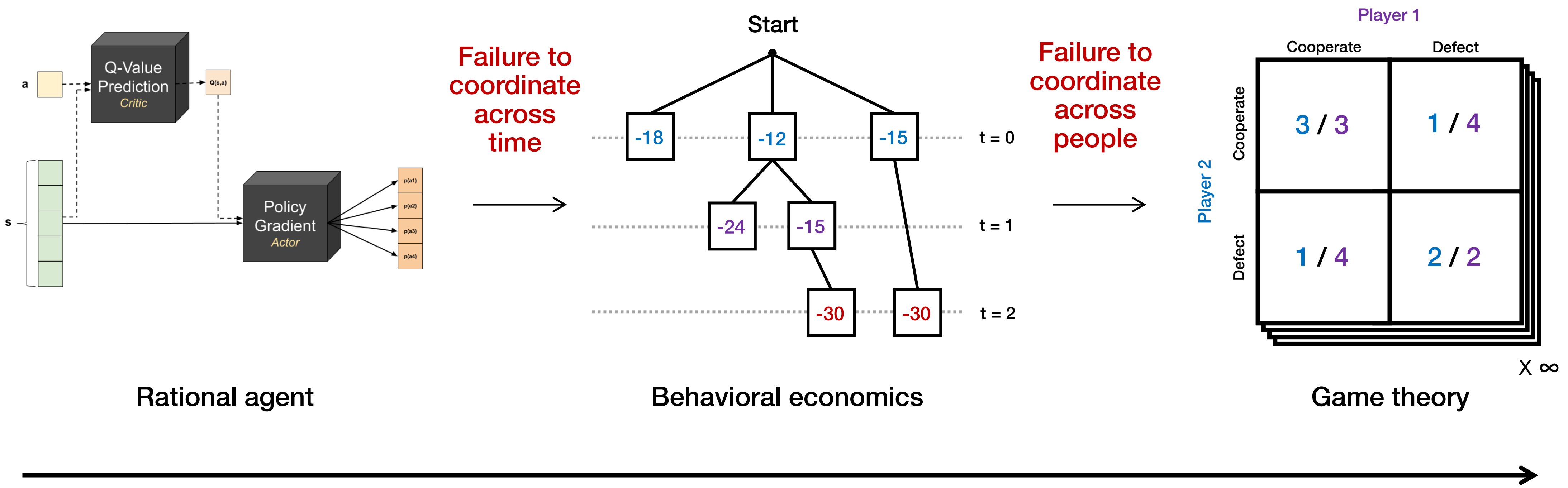
Networked games



People interact locally with different structural conditions

Interactions can break down mean-field descriptions

Complex social systems are often poorly described by aggregate statistics



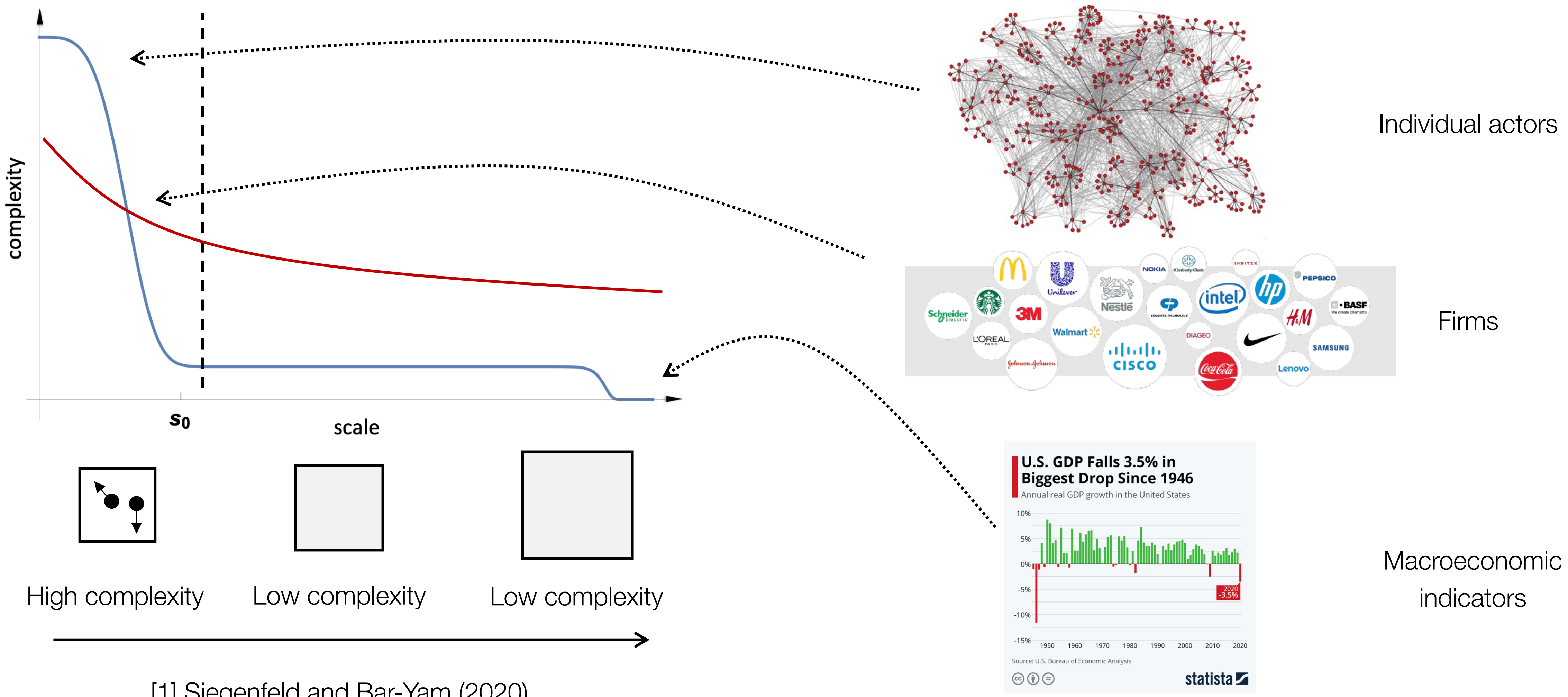
Rational agent — Behavioral economics — Game theory

Rational agent assumptions break down at larger levels of aggregation (individual optimal ≠ social optimal). You cannot simply take the means of their decisions!

Are markets efficient around equilibrium prices? Are companies and countries rational actors?

Theories of complex systems

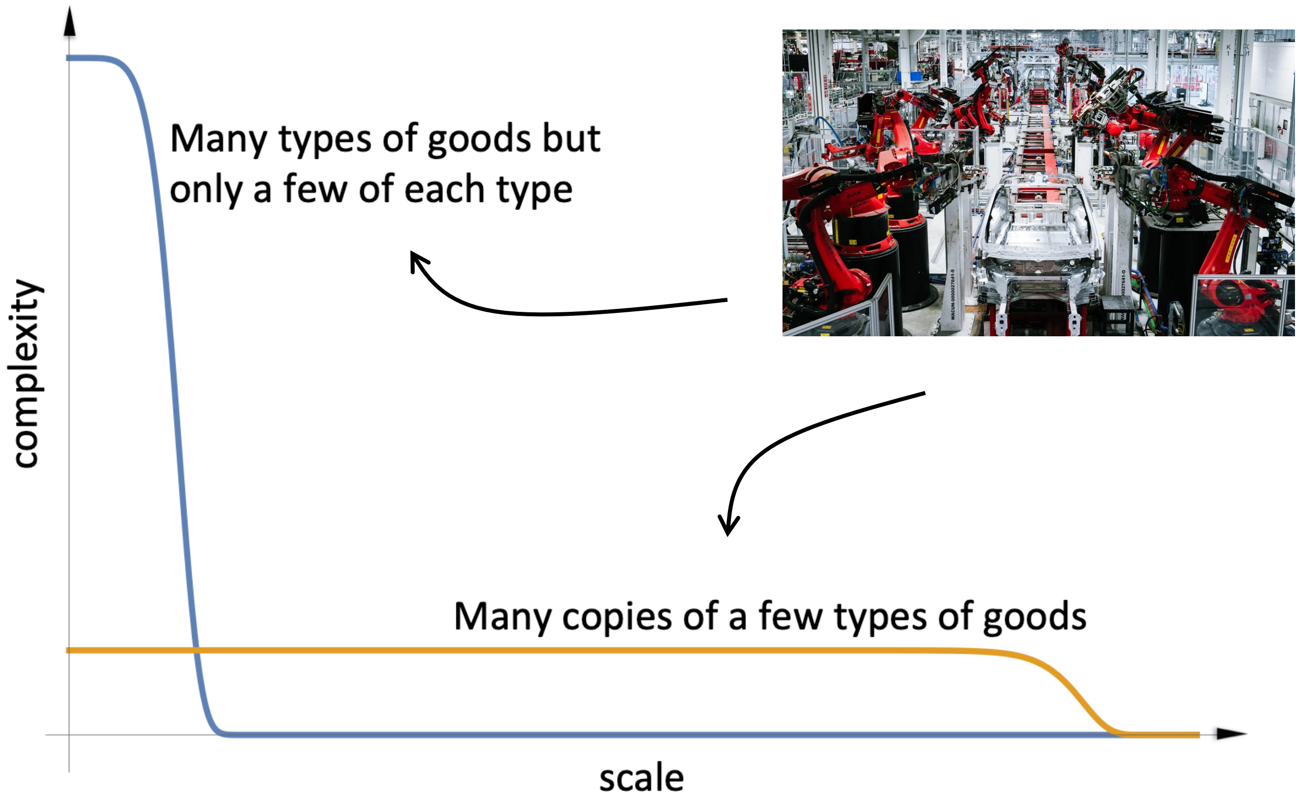
Looking at you, economics.



[1] Siegenfeld and Bar-Yam (2020)

Tradeoffs between complexity and scale

Complexity requires order: for there to be structure at larger scales, there must be coordination between many components at smaller scales. But this means that smaller scales are now limited by independencies.



[1] Siegenfeld and Bar-Yam (2020)

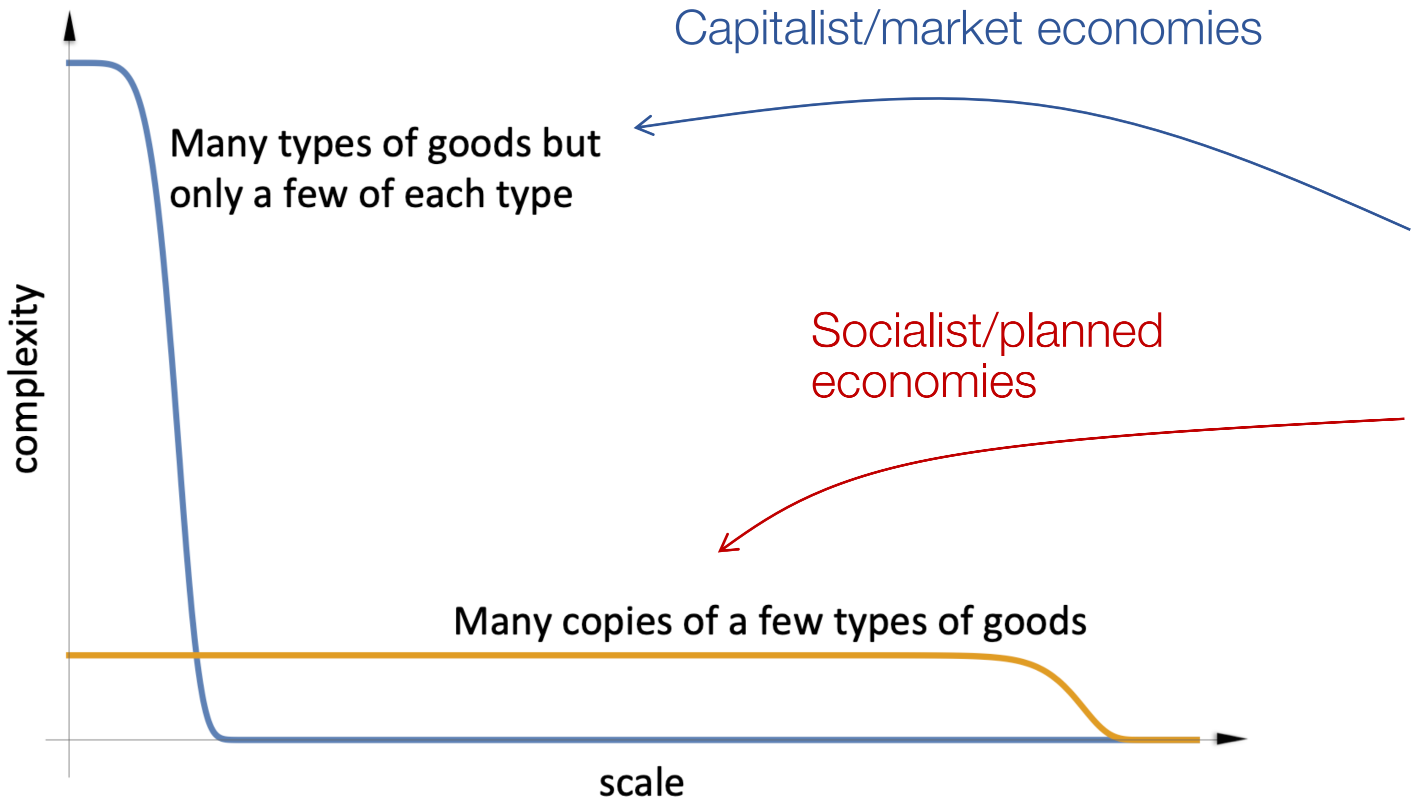
Tradeoff between adaptability and efficiency:

Adaptable: many independent actions at small scales. Inefficient but robust.

Efficient: many parts interact and work in concert, performing an optimal task at the largest scale. Efficient but fragile.

Tradeoffs between complexity and scale

Complexity requires order: for there to be structure at larger scales, there must be coordination between many components at smaller scales. But this means that smaller scales are now limited by independencies.



[1] Siegenfeld and Bar-Yam (2020)

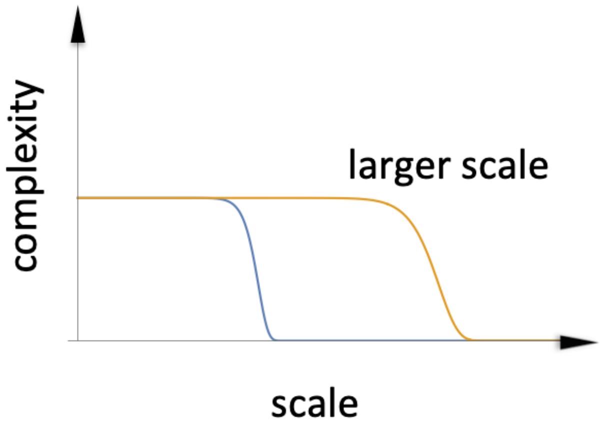
Tradeoff between adaptability and efficiency:

Adaptable: many independent actions at small scales. Inefficient but robust.

Efficient: many parts interact and work in concert, performing an optimal task at the largest scale. Efficient but fragile.

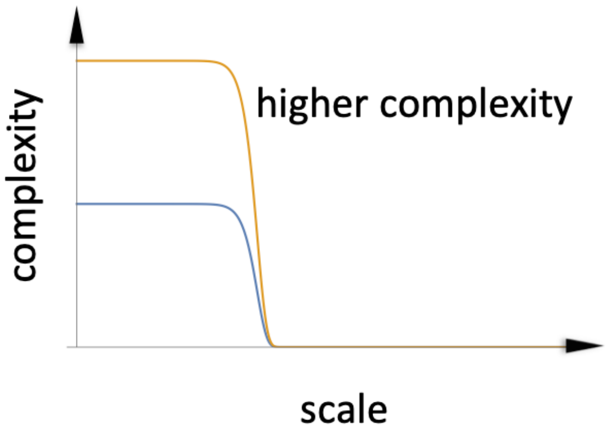
You can’t have your cake and eat it too, but sometimes, you need a particular system to solve certain problems.

Matching complexity profiles with the environment



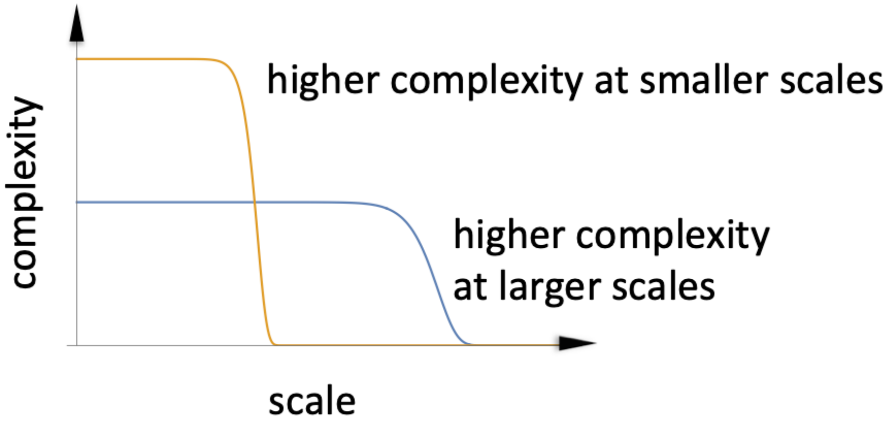
A smaller army vs. a larger army

Bigger army wins!



A less vs. more well-trained army

More trained army wins!



An insurgent army vs. a large national army

The battlefield environment decides the winner: fighting in the cities vs. in the open fields.

[1] Siegenfeld and Bar-Yam (2020)

Systems can adapt to their environments over time

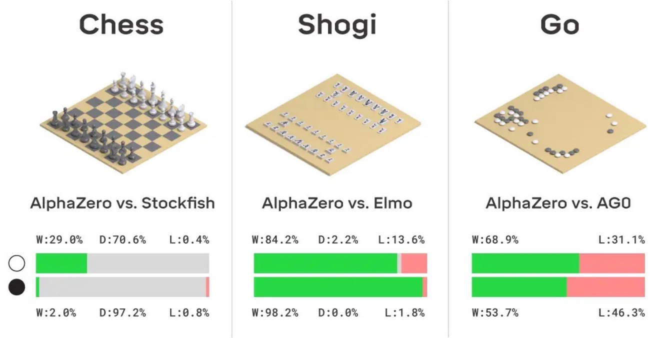
3.5 billion years of natural selection

Low-complexity organism

High-complexity organism

```
pi_net = nn.Sequential(
            nn.Linear(obs_dim, 64),
            nn.Tanh(),
            nn.Linear(64, 64),
            nn.Tanh(),
            nn.Linear(64, act_dim)
         )
```

Machine learning

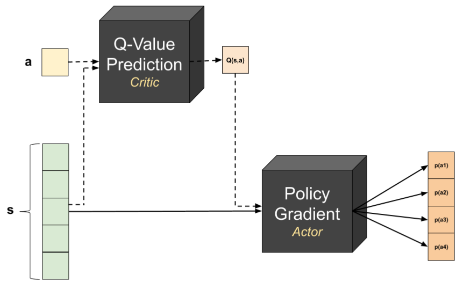


Low-complexity program

High-complexity program

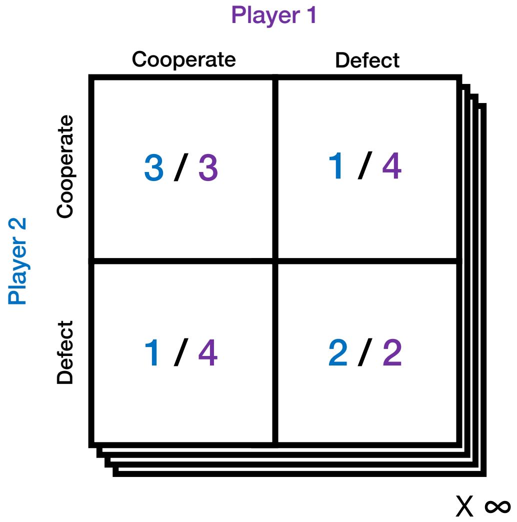
Multi-scale adaptation

Learning doesn't just happen via gradient descent!



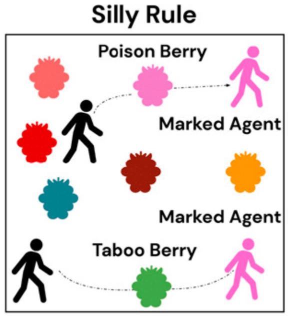
Optimizing across a lifetime

Habits
Knowledge



Optimizing across interactions

Norms
Institutions



Optimizing across generations

Culture
Morality

Optimizing across species

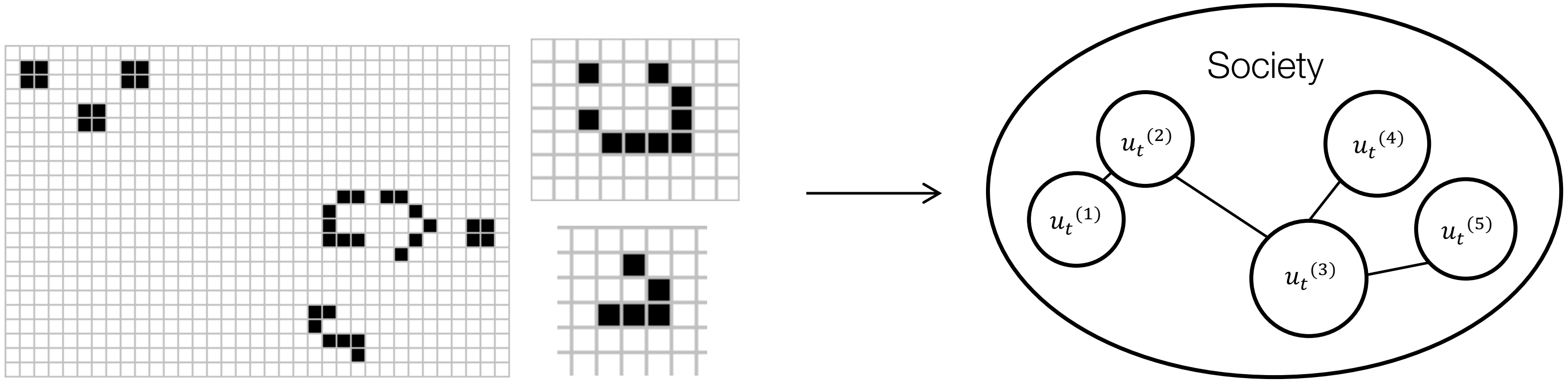
Biology
Behavorial traits

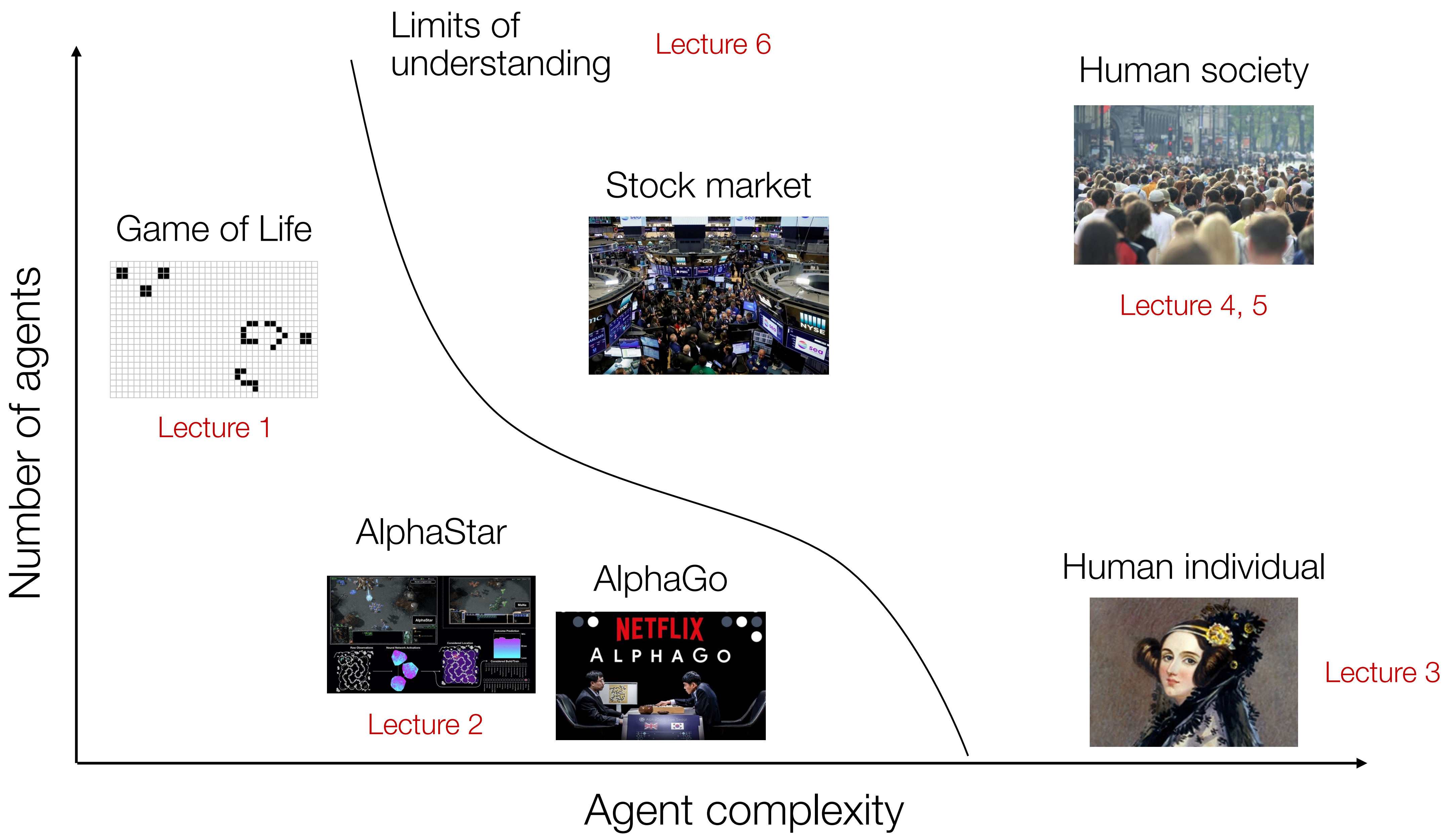
Our definition of a complex system

A complex system is made up of strongly interacting components that exhibit correlated behavior between scales – wherein microscopic details alter the collective dynamics of the system – such that its macroscopic properties cannot be easily described by a small number of aggregate measures.

In other words, it is a multi-agent system whose whole is greater than the sum of its parts.

Self-organized behavior




Limits of understanding
Lecture 6
Human society
Stock market
Lecture 4, 5
Game of Life
Number of agents
Lecture 1
AlphaStar
AlphaGo
NETFLIX
ALPHAGO
Human individual
Lecture 3
Lecture 2
Agent complexity

Parting words

Science and math can be beautiful, but they also provide powerful tools for us to create positive change in the world.

Use them well, use them to do things that matter to you!

References and additional resources

- An Introduction to Complex Systems Science and Its Applications: great non-mathematical review of complex systems
- Chaos: The Science of the Butterfly Effect video by Veritasium
- Dynamics of Complex Systems: review of many topics, more mathematically in-depth
- Statistical Mechanics: Entropy, Order Parameters, and Complexity: fun undergraduate-level textbook on statistical physics from a complexity perspective
- A Third Wave in the Economics of Climate Change: why complexity science is important for thinking about climate policy
- Systems Effects: Complexity in Political and Social Life: politics and complex systems
- How China Escaped the Poverty Trap: economic development and complex systems
- Courses from the Santa Fe Institute